Pitch-class set theory and helix model

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Abstract. The paper suggests a possible intersection between "pitch-class set theory" theorized by A. Forte and "helix model" proposed by M. W. Drobish. This intersection turns out to be an expansion of the concept of "pitch-class set", so that it can express additional information concerning the frequencies under consideration. This combination produces a useful tool, both in musical composition and in musical analysis. The paper reports in its conclusion an example of the use of such instrument by applying it to analyse some excerpts of the piece *Nunc* of Italian composer Goffredo Petrassi.

Keywords. Contemporary Music Composition, Music History, Music Theory

1 Introduction

Between 1964 and 1973, Forte develops the notion of "sets of pitch classes" or "pitch-class sets" [1]. These are collections of notes, which are considered independently from their octave placement. Forte found also relationships that connect the sets, thus allowing the confluence of multiple sets in "complexes" or "subcomplexes". The importance of Forte's theories is due to the ability to highlight, also in the atonal world, the internal consistency of musical structures and the links between the sections of a musical work [2].

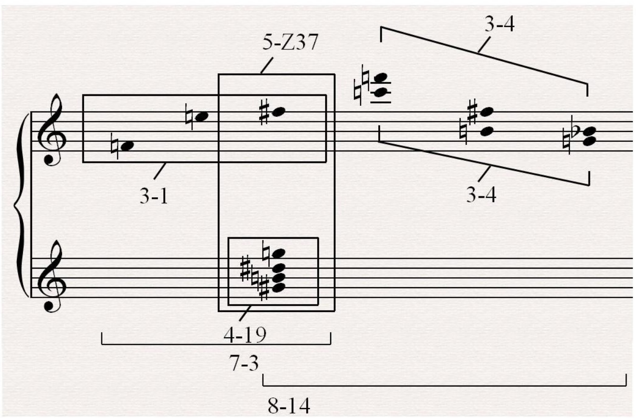

Fig. 1. Example of segmentation into sets and naming of the sets

In 1855 M. W. Drobish suggested the “helix model”, as a representation of the "pitch space". In this model the chromatic scale is distributed along a helix, and each pitch class is ideally placed on a straight line connecting it with the same pitch class in other octaves. The helix model is also represented by a cylinder with a "linear pitch space" wrapped around it. All pitches in octave relationship between them are placed along an ideal straight line here too [3]. According to the helix model, each pitch is composed of “pitch-class" (that indicates its position in the circle of pitch classes) and "brightness" or brilliance [4] (indicating its position on the vertical axis).

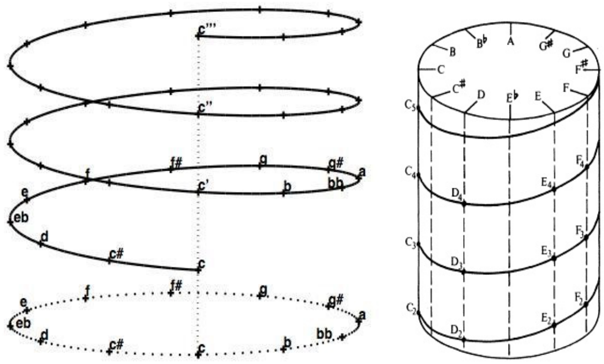

Fig. 2. Chromatic scale distributed along a helix and the cylinder with a linear pitch space

Being aware of the concepts of pitch class, pitch-class sets and set-theory analysis and related studies leading to the current perception of Forte's sets [5] and using Allen Forte's theory, one is left to wonder about the relationship that connects these and the vast area of timbre.
An integration of set-theory analysis with Drobish structures comes to mind, for example by characterizing the pitch classes according to their frequency positioning. An intersection between Forte studies and those of Drobish arises from the suggestion provided by the representation of the helix model. In this model, in fact, as already explained above, pitch classes are taken into consideration, but a single pitch class placed in different octaves is not considered an *unicum*, but it is differentiated and considered separately , being associated graphically at different points on the helix turns.

2 Methods

Each sound has its own spectrum and each musical note has its own harmonic spectrum, consisting of multiple frequencies components placed in harmonic relationship with each other and each with a given magnitude. The center of gravity of the spectrum or "spectral centroid" [6] of each musical note, is calculated as the sum of the product of each component frequency multiplied by its intensity, this until the last frequency component present in the spectrum, the sum thus obtained is divided by the sum of all intensities in the spectrum.

$$\mu_S = \frac{\sum_i f_i I_i}{\sum_i I_i}$$

Fig. 3. Calculation of the spectral centroid

The result is a frequency (measured in Hz) whose value is rescaled between 0 and 1 and it represents an index of "brightness" or of brilliance of that musical note. This calculation, with its index assignment, is applied to each element that constitutes a given "pitch-class set"; the index, during the musical note's existence will undergo several changes. In order to take these into account, an appropriate change to the representation of the index from a single numerical value to a continuous mathematical time function has been carried out. All these continuous time functions, each assigned to a constituent element of a given "pitch-class set" are then combined into a graph which shows the evolution of the "brightness" index of the whole "pitch-class set" in the example. Thus each "pitch-class set" can be represented by Forte's primary forms indicating the pitch class groupings within a chart.

3 Results

In order to illustrate the application of what has been hitherto discussed, you can consider some excerpts of the musical work *Nunc* for guitar, written in 1971 by composer Goffredo Petrassi (1904-2003). You can observe a set of five notes, shown in the following figure.

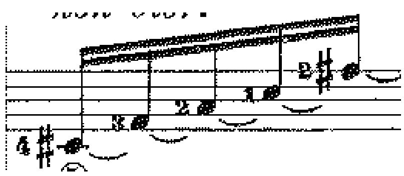

Fig. 4. A set from "Nunc" by G. Petrassi

This set has been initially identified by segmentation, and then the necessary operations have been applied to the constituent notes in order to find the corresponding primary form (5-22). Wanting to briefly explain the method for reconduct any set in one of the primary forms provided by Allen Forte, the steps to be taken are the following: the set of notes is initially placed so that the numbers that represent the pitch classes are in ascending order, then it is transposed on the pitch class "c" corresponding to the number zero, finally, with some operations, the smaller intervals between the pitch class components are placed in the left part of the set. The primary form 5-22 does not express the eighth of each constituent note. In Fig. 5 a graph of the brightness of the "pitch-class set" of Fig. 4 is proposed.

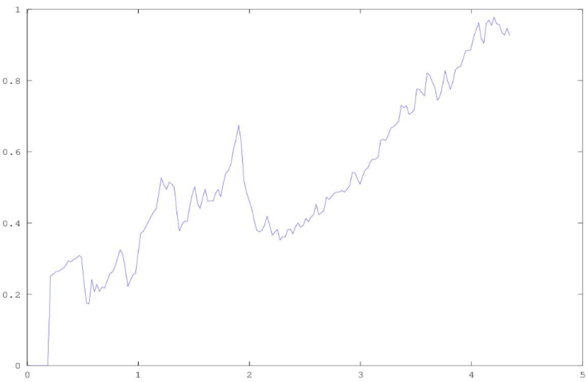

Fig. 5. Graph of brightness of the set of Figure 4

This type of representation can also detect other set characteristics. Continuing the observation of others sets from “*Nunc*" the same process used for the above set is repeated for the two "pitch-class sets" shown in Fig. 6.

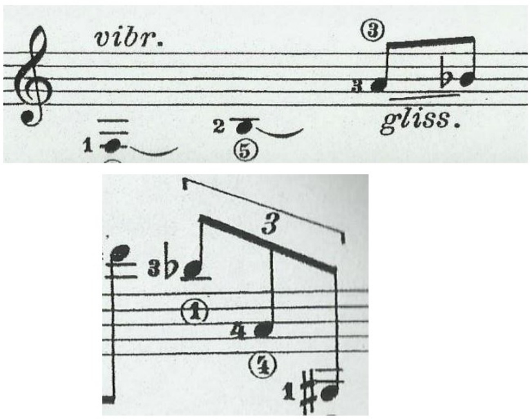

Fig. 6. Other sets from "Nunc" by G. Petrassi.

Both "pitch-class sets" can be referred to the same primary form (4-5). Fig. 7 shows a plot of the "brightness" of the first set of Fig. 6. Fig. 8 lots the "brightness" of the second set of Fig. 6.

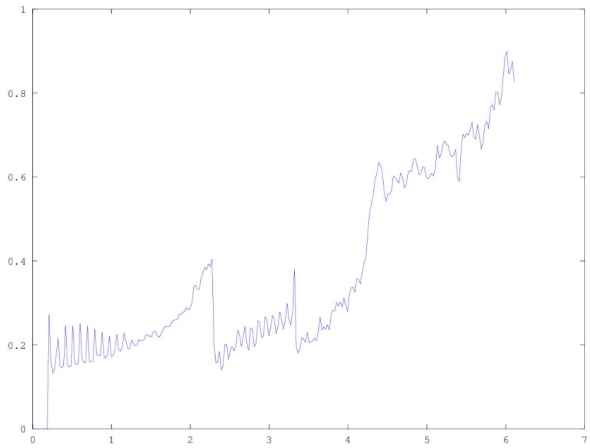

Fig. 7. Graph of brightness of the first set of Fig. 6

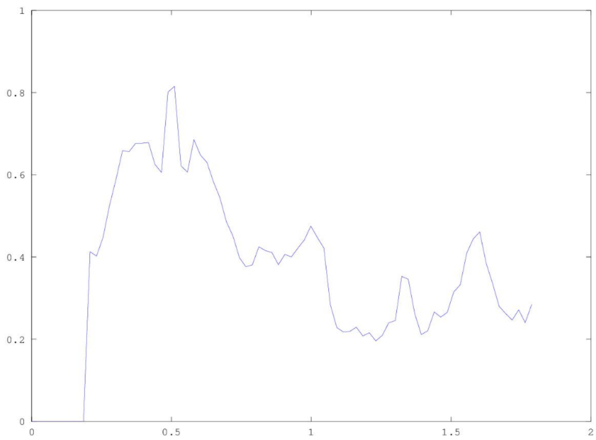

Fig. 8. Graph of brightness of the second set of Fig. 7

Fig. 9. Another set from "Nunc" by G. Petrassi

Note that, although the two sets can be referred to the same primary form, they are quite different when the brightness parameter is taken into consideration. In fact, the "brightness"

index of the first set of Fig. 6, shown in Fig. 7, indicates a trend characterized by an ascending movement. The "brightness" index of the second set of Fig. 6, illustrated in Fig. 7, consists of two areas, the first of higher values, above 0.5, with a downward movement reaching the second area below the threshold 0.5. In Fig. 9 you can observe another set of the same piece of Goffredo Petrassi, whose primary form appears to be the
4-4 along with the corresponding graph of its brightness. The graphs were made with Octave and the horizontal axis shows the duration of each extracted and analyzed fragment.

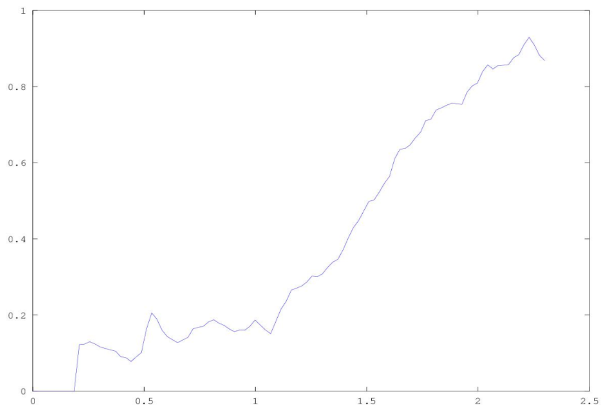

Fig. 10. Graph of brightness of the set of Fig. 10.

From the comparison between the three last sets, those of Fig. 6, 7 and 8 and those of the Fig. 9 and 10, it the similarity between the first set of Fig. 6 and Fig. 7, which has a 4-5 primary form, and the "pitch-class set" of Fig. 4 and 5 (5-22 primary form) is striking from the point of view of "brightness". In addition, a relationship between the second set of Fig. 6 (and Fig. 8) referable to the 4-5 primary form and "pitch-class set" of Fig. 9 and 10, referable to the 4-4 primary form, can be determined on the basis of the brightness parameter. This relationship can be assimilated to the result of a musical retrogradation, applied to the first set currently under consideration. In fact, it is easy to observe that both graphs relating to the brilliance of the two sets are characterized by an area above the threshold of 0.5 and an area between the values of 0 and 0.5. The two areas are connected by a downward movement in the graph of Fig. 8 and by an ascending movement in the graph of Fig. 10.

4 Discussion

The introduction of the "brightness" parameter into Allen Forte's set theory may be useful in compositional and analytical activities as it makes the brilliance of each set more clear and visible, giving the opportunity to appreciate not only the pitch classes in use, but also their frequency positioning. In addition, observing sets from the point of view of the brilliance may lead to a further classification of the sets, which will be able to be associated (in parallel to the constitution of "complexes" and "subcomplexes") and also compared according to the new parameter brightness.

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