Calculating Dissonance in Chopin's Étude Op. 10 No. 1

Nikita Mamedov and Robert Peck

Department of Music
nmamed1@lsu.edu

Abstract. The twenty-seven études of Frédéric Chopin are exemplary works that display the composer's poetic musical language, combining keyboard techniques, virtuosity, and artistic imagery. Étude Op. 10 No. 1 in C Major presents the pianistic technique of playing arpeggios, where a set of ascending and descending arpeggiated sequences span across multiple octaves. This study mathematically analyzes the dissonance level of this work, using the Interval Dissonance Rate (*IDR*) – a technique that integrates mathematical and musical analyses and measures the amount of dissonance in a piece of music, using modified interval-class vectors (v) and the frequency of recurrent pitches (p_n) to determine the percentage of dissonant (*DI*) and consonant (*CI*) intervals. There are three significant theoretical observations that can be deducted, based on the application of *IDR*. First, the *IDR* of the composition is 14.14%. Second, the B-section of the work (*IDR* of 16.16%) is the most dissonant and therefore, most tonally adventurous. Third, the *IDR* verifies that Chopin emphasizes the d_2 interval-class of the modified interval-class vector for dissonance, generating tension in harmonies by using major seconds, minor sevenths, and equivalent intervals.

Keywords. Chopin, dissonance, étude, interval-class

1 Introduction

Frédéric Chopin (1810-1849) is an important composer of the Romantic era, known for a vast compositional repertoire predominantly for piano, influenced by Polish culture, art, and literature [1]. Unlike his predecessors, such as Joseph Haydn and Ludwig van Beethoven, as well as his contemporary – Franz Liszt, Chopin establishes his compositional style early in his career and does not undergo any major stylistic changes [2]. Throughout his life, Chopin completed twenty-seven piano études, which include twelve works in Op. 10, twelve works in Op. 25, and *Trois Nouvelles Études* without an opus number [3]. This study dissects Étude Op. 10 No. 1 in C Major.

An étude is a type of composition that focuses on one or several specific pianistic challenges. In his études, Chopin merges advanced pianistic technique, virtuosity, and artistic imagery, all while preserving Romantic lyricism. In Op. 10 No. 1, Chopin presents the technique of playing arpeggios with the right hand, where a set of ascending and descending arpeggiated sequences span across multiple octaves. The work is in A-B-A^1 form and contains seventy-nine measures. Sections A and B are symmet-

rical; each is twenty-four measures in length.
This study focuses on the notion of musical dissonance, created between two or more harmonic structures. This research will provide mathematical proof for three theoretical observations. First, according to the Interval Dissonance Rate (*IDR*), Op. 10 No. 1 carries 14.14% of dissonance. Second, the middle section of the étude is the most dissonant and therefore, most tonally adventurous, when compared to other separate sections as well as the *IDR* of the piece as a whole. Third, the *IDR* verifies that Chopin emphasizes the d_2 interval-class for dissonance, generating tension in harmonies by using major seconds, minor sevenths, and equivalent intervals.

2 Interval Dissonance Rate

Musical dissonance is a significant characteristic in the stylistic understanding of Romantic music. Composers use dissonance to create a variety and musical contrast between two or more compositional segments. Dissonance is generated by tension among two or more harmonic structures. Chopin generates musical tension to create a necessity for resolution. The *IDR* is an analytical tool that integrates mathematical and musical analyses and measures the amount of dissonance in a piece of music, using modified interval-class vectors (v) and the frequency of recurrent pitches (p_n) to determine the percentage of dissonant (*DI*) and consonant (*CI*) intervals in semitones (t) [4]. The *IDR* is based on the following conditions:

$$\begin{aligned}
&DI = \{d_1, d_2, d_3, d_4\}; d_1 = 1t + 11t; \\
&d_2 = 2t + 10t; d_3 = 6t;\ d_4 = d_x(CI) \\
&CI = \{c_1, c_2, c_3, c_4\}; c_1 = 0t + 12t; \\
&c_2 = 3t + 9t; c_3 = 4t + 8t; c_4 = 5t + 7t \\
&IDR\%(x) = \frac{TI}{DI}, where\ TI \wedge DI\ = v(x) =\ icv\ +\ p_n
\end{aligned} \tag{1}$$

The interval-class vector (*icv*) is a string of digits that represents interval-classes, such as

$$t_a < icv < t_b \tag{2}$$

In traditional music theory, the interval-class vector contains six interval-classes, such as

$$< x_1, x_2, x_3, x_4, x_5, x_6 > \tag{3}$$

$<x_1>$ is the total amount of minor seconds, major sevenths, and equivalent intervals;
$<x_2>$ is the total amount of major seconds, minor sevenths, and equivalent intervals;
$<x_3>$ is the total amount of minor thirds, major sixths, and equivalent intervals;
$<x_4>$ is the total amount of major thirds, minor sixths, and equivalent intervals;
$<x_5>$ is the total amount of perfect fourth, perfect fifths, and equivalent intervals;

<x_6> is the total amount of tritones and equivalent intervals.

The interval-class vector contains a range of interval-classes between a minor second, where $t = 1$, and a major seventh, where $t = 11$. Therefore,

$$0 < icv < 12 \tag{4}$$

However, the interval-class vector does not recognize interval-classes that have a distance of $0t$, $12t$, and its equivalent. Since Romantic music is predominantly tonal, unisons, octaves, and similar intervals are frequently used to generate major and minor triads, and therefore, must be taken into consideration when calculating for dissonance. This situation is especially evident in the first étude, which is based on the major arpeggio. Traditionally, an arpeggio of a tonic triad consists of notes on scale degrees ^1, ^3, ^5, and ^8. Figure 1 shows a C Major arpeggio and the generation of interval-classes in the modified interval-class vector between pitches C, E, and G. These include one c_1 interval-class, one c_2 interval-class, two c_3 interval-classes, and two c_4 interval-classes.

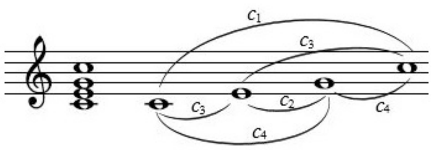

Fig. 1. C-Major arpeggio and the generation of c_1, c_2, c_3, and c_4 interval-classes.

A modified interval-class vector (v) is appropriate for *IDR* calculations. The modified interval-class vector contains a range of all possible distances between two or more notes, such as

$$0 \leq v \leq 12 \tag{5}$$

While other perspectives to working with consonance and dissonance exist, there are two reasons why a mathematical approach is the most suitable. First, it eliminates the notions of subjectivity. For instance, one's culture, musical training, musical apprehension, and historical evolution can decide whether an interval sounds consonant or dissonant, when analyzing from psychological perspective [5]. That is why certain intervals are recognized as dissonant at one historical epoch and as consonant at another. An example can be seen with the intervals of thirds and sixths before Walter Odington's theories on just intonation, and after [6].
Second, the *IDR* does not allow for diversity in the dissonance results. From an acoustical perspective, there exist non-constant variables that can influence the level of dissonance, such as the tuning systems [7]. For example, the Pythagorean tuning system is based on perfectly tuned fourths, fifths, and octaves, while the equal temperament tuning system is based on equal distribution of pitches between every two neighboring semitones [7]. Therefore, the distance between any two pitches will vary, resulting in different acoustical calculations of dissonance. This does not happen in

IDR calculations, since the *IDR* divides all intervals based on distance in semitones, creating a definite distinction between what is considered consonant and dissonant.

3 Étude Op. 10, No. 1

This composition serves as the prelude to the cycle of Chopin's first twelve études. Chopin completes the first étude in 1829 and publishes it in 1833 along with the other works of Op. 10 [1]. The first harmonic cycle appears in the opening nine measures of the work, starting and ending on C Major [8]. Figure 2 shows the opening phrase of the work with ascending and descending arpeggiated patterns in the right hand, seen in the perpetuum mobile motion of sixteenth notes and with an accompanimental support from the musical material in the left hand, seen in whole, half, and quarter notes.

Fig. 2. The first harmonic cycle (mm. 1-9) of Chopin's Étude Op. 10 No. 1.

It is possible to explain the pianistic technique of arpeggiation from theoretical perspective and Schenkerian analysis is the main theory that explains such notion from a standpoint of analytical elaborations. Schenkerian analysis defines multiple structural levels of a work, and establishes the analytical relationship between the harmony and the melody of the étude [9]. And while Schenkerian theory can be used to locate the dissonance, it does not calculate it. Figure 3 shows the Schenkerian analysis with *IDR* calculation of the opening harmonic cycle in the initial nine measures of the work. Rosen states that the musical idea is realized through the configuration of the hand and arm [2]. Such configurations are marked with arrows, representing the alternating ascending and descending arpeggio patterns that are responsible for the construction

of the whole compositions.
This cycle contains seven harmonies. Three of them are purely consonant, each with an *IDR* of 0.00%: the tonic C Major harmony in mm. 1-2, the subdominant F Major harmony in mm. 3, and a closing tonic C Major harmony in m. 9. Each of these harmonies are only based on the scale degrees ^1, ^3, and ^5. The other four harmonies contain dissonance and range in dissonance level from 8.70% to 45.00%. These include (in order of *IDR*%): the dominant seventh harmony with a descending bass line in m. 5 with an *IDR* of 8.70%, the applied dominant seventh chord in m. 6 with an *IDR* of 16.67%, the dominant seventh chord in mm. 7-8 with an *IDR* of 34.78%, and the applied half-diminished seventh chord in m. 4 with an *IDR* of 45.00%. The *IDR* explains the tonal notion of the initial cycle of this étude. The cycle begins with two consonant harmonies before entering a dissonant zone in mm. 4-8, necessary to create enough tension for return towards the tonic in m. 9.

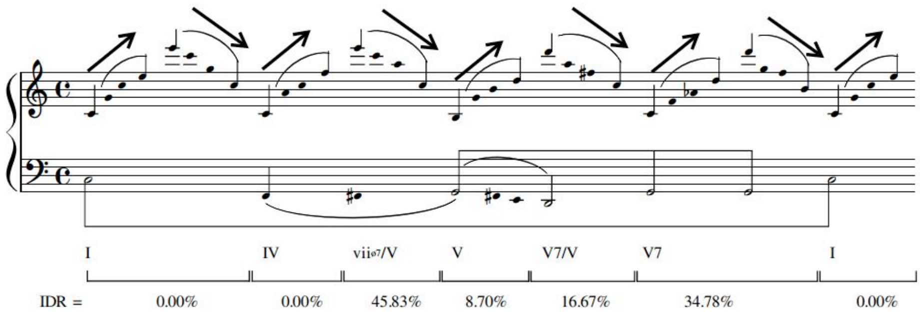

Fig. 3. The first harmonic cycle (mm. 1-9) of Chopin's Étude Op. 10 No. 1.

4 Final Results

The *IDR* analysis generates the following data, as shown in Table I.

TABLE I: THE *IDR* OF CHOPIN'S OP. 10 NO. 1.

Op. 10 No. 1	*IDR%*	*c1*	*c2*	*c3*	*c4*	*d1*	*d2*	*d3*	*d4*	*σ (TI)*
Whole Composition	14.14	47.87	6.24	13.21	18.55	2.12	7.02	4.12	0.87	15.45
Section A (bars 1-24)	11.88	46.81	7.62	13.12	20.57	1.42	4.96	4.61	0.89	15.32
Section B (bars 25-48)	16.16	50.00	5.08	12.70	16.06	1.93	10.16	3.25	0.81	16.09
Section A1 (bars 49-79)	14.55	47.19	5.93	13.65	18.68	2.85	6.45	4.35	0.90	15.20
σ (A, B, A1)	2.16	1.74	1.30	0.48	2.27	0.73	2.68	0.72	0.05	

The *IDR* of the whole composition is 14.14%. The main reason for such low dissonance rate is the predominance of c_1 interval-class, amalgamating unisons and octaves that occurs 47.87% of the time. This can be noticed in the left hand of the étude that contains doubled notes throughout the whole work. Furthermore, the B-section of the étude is the most dissonant, when comparing to sections A and A^1. We can therefore conclude that the B-section is the most tonally adventurous, where Chopin circulates

through closely related and distant keys, yet only returning to C-Major at the beginning of section A^1. Likewise, the *IDR* analysis verifies that Chopin particularly emphasizes on d_2 interval-class for dissonance, which allows the composer to generate tension in harmony by using intervals, such as major seconds, minor sevenths, and equivalent. As seen from Table I, the d_2 interval-class occurs 7.02% of the time throughout the work, while d_1 and d_3 occur 2.12% and 4.12% respectively. The uniqueness of d_2 interval-class is especially seen in the B-section, occurring 10.16% of the times, while the combination of d_1 and d_3 interval-classes provide 5.18% of the dissonance.
The standard deviation allows one to see the changes that occur between the interval-classes throughout the whole work and the changes that occur in each interval-class, based on the sectional analysis of the étude. From the data, it is evident that the standard deviation of all interval-classes of each section does not alter much; the standard deviation of total intervals varies between 15.20 and 16.09. This means that for the most part, the distribution of the interval-classes in each section is similar. However, the distribution of each interval-class throughout the work is not. For instance, the c_3 interval-class contains the standard deviation of 0.48. In each section, c_3 is seen between 12.70% and 13.65%. On the other hand, the biggest distribution occurs in the d_2 interval-class, which is seen between 4.96% and 10.16%.

5 Conclusion

The *IDR* analysis allows one to mathematically calculate the amount of dissonance in a piece of music. Based on these results, it is evident that even a tonally consonant composer will incorporate dissonance in his compositions. While Étude Op. 10 No. 1 is based on arpeggiation, musical elements, such as various seventh chords, passing and neighbor tones, dominant substitutions, and chromaticism will increase the dissonance level of the work. The *IDR* allows to verify this from empirical point of view.

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