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Chord Progression Analysis by Labelled Lambek Calculus

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Abstract. Music and language are thought to share a common origin, leading to numerous research endeavors that have sought to analyze music through a compositional approach, grounded in linguistic grammar rules. In this study, we extend and refine this method, demonstrating that the analysis can be systematically represented using rigorous proof theory, akin to how formal logic elucidates natural language semantics. Our approach involves the use of sequent calculus to construct a proof, notably extending Lambek calculus for categorial grammar to a labeled version. The introduction of labels enables each term within a sequent to be interpreted as a chord with a specified key (tonality). This labeling allows us to precisely determine the grammatical validity of a given sequence of chords. Given that music grammar varies with genre and era, our formalism is designed to accommodate flexible addition and reduction of applicable rules. Additionally, we show that this process of analysis is reversed in Tableaux style, to simulate the process of composition. This methodology not only enhances our understanding of music from a linguistic perspective but also provides a versatile framework for adapting to the evolving nature of musical grammar.

Keywords. Music Analysis, Lambek Calculus, Proof Theory, Tableaux, Chord Progression.

1 Introduction

As Charles Darwin mentioned, the origin of human language is said to be one and the same as music [15], which must have been employed for wooing between primordial males and females. Since our natural languages are classified approximately as context-free language (CFL) in Chomsky hierarchy, the syntax of a sentence is structured in tree; thus, if the origin is common music, too, should be structured in a hierarchical tree.

Thus far, Generative Theory of Tonal Music (GTTM) [8], Rohmeier's Generative Synax Model (GSM) [12], and so on, have shown such tree-structured analyses of mu-

sic, and among such trials Steedman et al. [5] has shown a construction by combinatorial category grammar (CCG). In this paper, we formalize music analysis in a tree style, extending beyond syntactic analysis to include formal semantics. Just as language semantics are represented in logical formalism, we explore interpretation through logic. Our goal is to analyze a music piece, given a sequence of chord names, and determine if a cadential structure can be derived. We treat a sequence of chord names as a syntactic structure, interpreting each chord name as semantics - a pair of a key and a degree. Subsequently, we compose these interpretations in a structural manner, ultimately reducing them to a cadence.
This paper is organised as follows: in Section 2, we set up our formalism. In the following Section 3, we'll show an analysis of "Stella by Starlight". In Section 4, we show that the analysis process can also be viewed as its dual, i.e., Tableaux, Finally, we summarise our contribution and limitations and discuss future issues.

2 Labelled calculus for music analysis

Chord Notation and Lexicon

First, we strictly distinguish the three kinds of chord notations.

Berklee Chord Names - Em7, A7, Cm7, etc… each of which is a set of diatonic notes, and are shown in upright fonts.

Key : Degree - A pair of a key (tonality) and a degree in roman numeric is shown by being connected by colon (:), e.g., C:I, G:iii, a:i, etc. The lowercase in key is a minor, and the lowercase numeric is a minor third, and they are shown in italic fonts.

Chord functions - T (tonic), and **D** (dominant), **S** (subdominant), shown in boldface fonts.

We provide a *Lexicon*, where a Berklee chord is looked up and it can be interpreted in multiple ways, as follows.

$$\begin{array}{lll}
\hline
\mathrm{F} & \Rightarrow & C:\mathrm{IV},\ F:\mathrm{I},\ B\flat:\mathrm{V},\cdots \\
\mathrm{G} & \Rightarrow & G:\mathrm{I},\ D:\mathrm{IV},\ a:\mathrm{VII},\cdots \\
\mathrm{B}\flat & \Rightarrow & F:\mathrm{IV},\cdots \\
\mathrm{C7} & \Rightarrow & F:\mathrm{V7},\cdots \\
\vdots & & \vdots \\
\hline
\end{array}$$

Lambek Calculus (LC) is a sequent calculus for Categorial Grammar (CG). Since CG can bind an adjacent word or a category either from the left-hand or the right-hand side, we need to provide rules in two different directions: $\rightarrow$ and $\leftarrow$[1]. In Kripke semantics of modal logic, $\square x$ means the necessity; that is, in all the accessible possible worlds, x must hold, and $\Diamond x$ means that there exists some possible worlds in which x holds. In we show a set of general rules for labelled Lambek calculus (LLC)[2], apart from the music construction.

Table 1 Labelled Lambek Calculus, where the right-hand side of '⊢' is intuitionistically restricted to only one term. (□) tells that if αRβ deduces β : x then α : □x, and (◊) vice versa. Note that αRβ appears order-free in sequents.

$$\frac{\Gamma \vdash \alpha : x \qquad \Delta, \alpha : y, \Sigma \vdash \beta : z}{\Delta, \alpha : y \leftarrow x, \Gamma, \Sigma \vdash \beta : z}\ (\leftarrow_L) \qquad \frac{\Gamma, \alpha : x \vdash \alpha : y}{\Gamma \vdash \alpha : y \leftarrow x}\ (\leftarrow_R)$$

$$\frac{\Gamma \vdash \alpha : x \qquad \Delta, \alpha : y, \Sigma \vdash \beta : z}{\Delta, \Gamma, \alpha : x \rightarrow y, \Sigma \vdash \beta : z}\ (\rightarrow_L) \qquad \frac{\alpha : x, \Gamma \vdash \alpha : y}{\Gamma \vdash \alpha : x \rightarrow y}\ (\rightarrow_R)$$

$$\frac{\Gamma \vdash \alpha : x \qquad \alpha : x \vdash \beta : y}{\Gamma \vdash \beta : y}\ (\text{Cut}) \qquad \frac{\Gamma \vdash \alpha : \square x}{\alpha R \beta,\ \Gamma \vdash \beta : x}\ (\square)$$

$$\frac{\alpha R \beta,\ \Gamma \vdash \beta : x}{\Gamma \vdash \alpha : \Diamond x}\ (\Diamond)$$

LLC rules for cadences

Hereafter, we use α, β, … for keys, and x, y, … for degrees. Now, we employ LLC for cadence rules. We regard αRβ is a key modulation from α to β, that is, key β is accessible from α if they are related. For example, if R represents a shift to the parallel key, chord α : x becomes β : y in the parallel key. R works in a different way, according to shifts to parallel, relative, dominant, and sub-dominant keys.

$$\alpha : x, \alpha R \beta \Rightarrow \beta : R(x)$$

together with other initial sequents for chord functions:

$$\alpha : I \vdash \alpha : \boldsymbol{T}, \boldsymbol{\alpha} : V \vdash \alpha : \boldsymbol{D}, \alpha : IV \vdash \alpha : \boldsymbol{S} \ldots$$

[1] In the original LC, /’ and \’ are used for two directions, but /’ is confusing in the context of chord representation (/’ is used for *doppel*-dominant or to denote a bass in a chord). Therefore, we employ arrows instead.

[2] The original LC includes a construction by dot (·), however, we omit them here since they are not mentioned in this paper.

We introduce Cadence Rules below, instantiating the labelled Lambek rules in Table

$$\frac{\Gamma \vdash \alpha : \mathbf{D} \qquad \Delta, \alpha : \mathrm{I} \vdash \alpha : \mathbf{T}}{\Delta, \Gamma, \alpha : \mathbf{D} \to \mathrm{I} \vdash \alpha : \mathbf{T}}\ (\to_L)$$

$$\frac{\Gamma \vdash \alpha : \mathbf{S} \qquad \Delta, \alpha : \mathrm{V} \vdash \alpha : \mathbf{D}}{\Delta, \Gamma, \alpha : \mathbf{S} \to \mathrm{V} \vdash \alpha : \mathbf{D}}\ (\to_L)$$

$$\frac{\alpha : \mathrm{I} \vdash \alpha : \mathbf{T} \qquad \Gamma, \alpha : \mathrm{I} \vdash \alpha : \mathbf{T}}{\Gamma, \alpha : \mathbf{T} \leftarrow \mathrm{I}, \alpha : \mathrm{I} \vdash \alpha : \mathbf{T}}\ (\leftarrow_L)$$

From the given initial sequents and instantiated rules, the right-hand side of '⊢' is always a chord function; it means that a sequence of chords is interpreted to bear a function. For an easy example, given an input chord sequence: G–C–D–G, we can obtain the following construction, consulting the lexicon.

$$\frac{\dfrac{\mathrm{G}\Rightarrow G{:}\mathrm{I}}{G : \mathrm{I} \vdash G : \mathbf{T}} \qquad \dfrac{\dfrac{\dfrac{\mathrm{C}\Rightarrow G{:}\mathrm{IV}}{G : \mathrm{IV} \vdash G : \mathbf{S}} \quad \dfrac{\mathrm{D}\Rightarrow G{:}\mathrm{V}}{G : \mathrm{V} \vdash G : \mathbf{D}}}{G : \mathbf{S} \to \mathrm{V}, G : \mathrm{V} \vdash G : \mathbf{D}}\ (\to_L) \quad \dfrac{\mathrm{G}\Rightarrow G{:}\mathrm{I}}{G : \mathrm{I} \vdash G : \mathbf{T}}}{G : \mathbf{D} \to \mathrm{I}, G : \mathrm{I} \vdash G : \mathbf{T}}\ (\to_L)}{G : \mathbf{T} \leftarrow \mathrm{I}, G : \mathrm{I} \vdash G : \mathbf{T}}\ (\leftarrow_L)$$

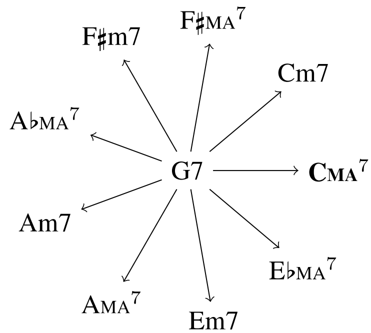
However when the chord C or D occupy longer time duration, we should regard they are local modulations as C:I or D:I. In which case, the latter part of a valid proof tree is constructed, as follows.

$$\frac{\dfrac{\dfrac{\mathrm{C}\Rightarrow C{:}\mathrm{I} \qquad C : \mathrm{I},\ {}_{C}R_{G} \Rightarrow G : \mathrm{IV}}{G : \mathrm{IV} \vdash G : \mathbf{S}} \qquad \dfrac{\mathrm{D}\Rightarrow D{:}\mathrm{I} \qquad D : \mathrm{I},\ {}_{D}R_{G} \Rightarrow G : \mathrm{V}}{G : \mathrm{V} \vdash G : \mathbf{D}}}{G : \mathbf{S} \to \mathrm{V}, G : \mathrm{G} \vdash G : \mathbf{D}} \qquad \dfrac{\mathrm{G}\Rightarrow G{:}\mathrm{I}}{G : \mathrm{I} \vdash G : \mathbf{T}}}{G : \mathbf{D} \to \mathrm{I}, G : \mathrm{I} \vdash G : \mathbf{T}}$$

3 Analysis of "Stella By Starlight"

The main reason for opting for the Labelled Sequent Calculus is the challenging nature of analyzing songs like 'Stella by Starlight' using the instruments introduced in [1]. This challenge stems from the absence of consequent cadences, i.e., that the cadences do not adhere to a strict order, as conventional proof theory rules need for finding a solution. Instead, it is evident that a cadence can resolve one, two, or more measures away from a given point.

Another reason is that encountering a dominant doesn't guarantee a strict resolution to its tonic; instead, it can lead to multiple potential resolutions:



Labelled sequents are helpful to take trace of the current tonality, something that Proof Theory alone couldn't do. To do something like that we will try to use a modified version of labelled sequents, introduced firstly in [4].

As an example, we take "Stella by Starlight" (Figure 1), which is very useful be- cause it contains numerous problematic cadences. For the sake of readability, the har- monic analysis is divided into multiple parts instead of being presented as one very long derivation. To better understand the example, we will describe each part of it.

A. These chords are extracted from measures 5–7, illustrating a classic cadence on E♭.

B. This represents a perfect cadence on B♭MA7, which can be easily deduced.

C. It is the combination of **A** and **B** denoted by $^{(*1)}$ and $^{(*2)}$. In the left section (mm. 3-4), a cadence occurs, but it is not resolved in the subsequent chords. In- stead, resolution happens on the B♭ in measure 9. This brings closure, resolving everything on the tonic of B♭. This kind of cadence is not an exception in music; it is something really common. As a metaphor, we can say that just as in speech where we might start discussing a focus and then change, only to finally come back to the initial topic, in music, cadences like these are frequent. The LLC here fits perfectly, helping us to understand when a certain cadence closes.

D. It is perfect cadence on Dm.

E. This is the union of all the preceding derivations, providing closure to each cadence. The unresolved cadence from the first two measures finds resolution in measure 11, the final measure of this structure, concluding all preceding derivations.

F. From here, we will not write every cadence in all the sequents, as the sequences are now easily closed. This is, in fact, a perfect cadence that resolves from the seventh flat degree.

G. This represents a minor cadence resolving on a seventh chord. Although it may seem a bit complex, the underlying concept is straightforward. The tree indicates an accessible relationship between D minor and D major, simplifying the solution.

H. Follow the same procedure as in **G**.

I. Derive a perfect cadence from the last chord in **H**.

L. A perfect cadence that passes from the seventh degree instead of the fifth.

M. N. O. These are all perfect cadences that are chained because the final chord of **M** is the first of **N** and the final chord of **N** is the first chord of the last cadence of the song in **O**.

Fig. 1. Stella by Starlight, written by Victor Young.

$$\boxed{\text{A}}\ \dfrac{\dfrac{\dfrac{\text{Fm7} \Rightarrow \text{E}\flat : \mathbf{II}}{\text{E}\flat : II \vdash \text{E}\flat : \mathbf{S}} \qquad \dfrac{\text{B}\flat 7 \Rightarrow \text{E}\flat : V}{\text{E}\flat : V \vdash \text{E}\flat : \mathbf{D}}}{\text{E}\flat : \mathbf{S} \to V, \text{E}\flat : V \vdash \text{E}\flat : \mathbf{D}}\,(\to_L) \qquad \dfrac{E\flat\,\text{MA}^7 \Rightarrow E\flat : \mathbf{I}}{\text{E}\flat : I \vdash \text{E}\flat : \mathbf{T}}}{{}^{(*1)}\text{E}\flat : \mathbf{D} \to I, \text{E}\flat : I \vdash \text{E}\flat : \mathbf{T}}\,(\to_L)$$

$$\boxed{\text{B}}\ \dfrac{\dfrac{\text{A}\flat 7 \Rightarrow \text{B}\flat : VII\flat}{\text{B}\flat : VII\flat \vdash \text{B}\flat : \mathbf{D}} \qquad \dfrac{\text{B}\flat\,\text{MA}^7 \Rightarrow \text{B}\flat : V}{\text{B}\flat : I \vdash \text{B}\flat : \mathbf{T}}}{{}^{(*2)}\text{B}\flat : \mathbf{D} \to I, \text{B}\flat : I \vdash \text{B}\flat : \mathbf{T}}\,(\to_L)$$

$$\boxed{\text{C}}\ \dfrac{\dfrac{\dfrac{\text{Cm7} \Rightarrow \text{B}\flat : II}{\text{B}\flat : II \vdash \text{B}\flat : \mathbf{S}} \qquad \dfrac{\text{F7} \Rightarrow \text{B}\flat : V}{\text{B}\flat : V \vdash \text{B}\flat : \mathbf{D}}}{\text{B}\flat : \mathbf{S} \to V, \text{B}\flat : V \vdash \text{B}\flat : \mathbf{D}}\,(\to_L) \qquad (*1) \qquad (*2)}{{}^{(*3)}\ \text{B}\flat : V, \text{E}\flat : \mathbf{D} \to I, \text{E}\flat : I, \text{B}\flat : \mathbf{D} \to I, \text{B}\flat : I \vdash \text{E}\flat : \mathbf{T}, \text{B}\flat : \mathbf{T}}\,(\to_L)$$

$$\boxed{\text{D}}\ \dfrac{\dfrac{\dfrac{\text{Em7}^{(\flat 5)} \Rightarrow \text{d}: II}{\text{d}: II \vdash \text{d}: \mathbf{S}} \qquad \dfrac{\text{A7} \Rightarrow \text{d}: V}{\text{d}: V \vdash \text{d}: \mathbf{D}}}{\text{d}: \mathbf{S} \to V, \text{d}: V \vdash \text{d}: \mathbf{D}}\,(\to_L) \qquad \dfrac{\text{Dm} \Rightarrow \text{d}: I}{\text{d:I} \vdash \text{d}: \mathbf{T}}}{{}^{*(4)}\text{d}: \mathbf{D} \to I, \text{d}: I \vdash \text{d}: \mathbf{T}}\,(\to_L)$$

$$\boxed{\text{E}}\ \dfrac{\dfrac{\dfrac{\text{Em7}^{\flat 5} \Rightarrow \text{d}: II}{\text{d}: II \vdash \text{d}: \mathbf{S}} \qquad \dfrac{\text{A7} \Rightarrow \text{d}: V}{\text{d}: V \vdash \text{d}: D}}{\text{d}: \mathbf{S} \to V, \text{d}: V \vdash \text{d}: \mathbf{D}}\,(\to_L) \qquad (*3) \qquad (*4)}{\text{d}: V, \text{B}\flat : V, \text{E}\flat : \mathbf{D} \to I, \text{E}\flat : I, \text{B}\flat : \mathbf{D} \to I, \text{B}\flat : I, \text{d}: \mathbf{D} \to I, \text{d}: I \vdash \text{E}\flat : \mathbf{T}, \text{B}\flat : \mathbf{T}, \text{d}: \mathbf{T}}\,(\to_L)$$

$$\boxed{\mathrm{F}}\quad \dfrac{\dfrac{\dfrac{\mathrm{B}\flat m7 \Rightarrow \mathrm{F}\text{: } iv}{\mathrm{F}\text{: } iv \vdash \mathrm{F}\text{: } \mathbf{S}} \qquad \dfrac{\mathrm{E}\flat 7 \Rightarrow \mathrm{F}\text{: } VII\flat}{\mathrm{F}\text{:} VII\flat \vdash \mathrm{F}\text{: } \mathbf{D}}}{\mathrm{F}\text{: } \mathbf{S} \to VII\flat, \mathrm{F}\text{: } VII\flat \vdash F : \mathbf{D}}\,(\to_L) \qquad \dfrac{\mathrm{F}_{\mathrm{MA}}{}^{7} \Rightarrow \mathrm{F}\text{: } I}{\mathrm{F}\text{: } I \vdash \mathrm{F}\text{: } \mathbf{T}}}{\mathrm{F}\text{: } \mathbf{D} \to I, \mathrm{F}\text{: } I \vdash \mathrm{F}\text{: } \mathbf{T}}\,(\to_L)$$

$$\boxed{\mathrm{G}}\quad \dfrac{\dfrac{\dfrac{\mathrm{Em7}^{(\flat 5)} \Rightarrow \mathrm{d}\text{: } II \qquad \mathrm{d}\text{: } II, {}_{d}\mathrm{R}_{D} \Rightarrow \mathrm{D}\text{:II}}{\mathrm{D}\text{:} II \vdash \mathrm{D}\text{: } \mathbf{S}} \qquad \dfrac{A7 \Rightarrow \mathrm{D}\text{: } V}{\mathrm{D}\text{: } V \vdash \mathrm{D}\text{: } \mathbf{D}}}{\mathrm{D}\text{: } \mathbf{S} \to V, \mathrm{D}\text{: } V \vdash \mathrm{D}\text{: } \mathbf{D}}\,(\to_L) \qquad \dfrac{\mathrm{D7} \Rightarrow \mathrm{D}\text{: } I}{\mathrm{D}\text{: } I \vdash \mathrm{D}\text{: } \mathbf{T}}}{{}^{*(4)}\mathrm{D}\text{: } \mathbf{D} \to I, \mathrm{D}\text{: } I \vdash \mathrm{D}\text{: } \mathbf{T}}\,(\to_L)$$

$$\boxed{\mathrm{H}}\quad \dfrac{\dfrac{\dfrac{\mathrm{Am7}^{(\flat 5)} \Rightarrow \mathrm{g}\text{: } II \qquad \mathrm{g}\text{: } II, {}_{g}\mathrm{R}_{G^{(\sharp 5)}} \Rightarrow \mathrm{G}^{(\sharp 5)} : II}{\mathrm{G}^{(\sharp 5)} : II \vdash \mathrm{G}^{(\sharp 5)} : \mathbf{S}} \qquad \dfrac{D7 \Rightarrow \mathrm{G}^{(\sharp 5)} : V}{\mathrm{G}^{(\sharp 5)} : V \vdash \mathrm{G}^{(\sharp 5)} : \mathbf{D}}}{\mathrm{G}^{(\sharp 5)} : S \to V, \mathrm{G}^{(\sharp 5)} : V \vdash \mathrm{G}\text{: } \mathbf{D}}\,(\to_L) \qquad \dfrac{\mathrm{G}^{(\sharp 5)} \Rightarrow \mathrm{G}^{(\sharp 5)} : I}{\mathrm{G}^{(\sharp 5)} : I \vdash \mathrm{G}^{(\sharp 5)} : \mathbf{T}}}{\mathrm{G}^{(\sharp 5)} : \mathbf{D} \to I, \mathrm{G}^{(\sharp 5)} : I \vdash \mathrm{G}^{(\sharp 5)} : \mathbf{T}}\,(\to_L)$$

$$\boxed{\mathrm{I}}\quad \dfrac{\dfrac{\mathrm{G}^{(\sharp 5)} \Rightarrow \mathrm{G}^{(\sharp 5)} : I \qquad \mathrm{G}^{(\sharp 5)} : I, {}_{G^{(\sharp 5)}}\mathrm{R}_{c} \Rightarrow \mathrm{c}\text{: } V}{\mathrm{c}\text{: } V \vdash \mathrm{c}\text{: } \mathbf{D}} \qquad \dfrac{\mathrm{Cm7} \Rightarrow \mathrm{c}\text{:} I}{\mathrm{c}\text{:I} \vdash \mathrm{c}\text{:} \mathbf{T}}}{\mathrm{c}\text{: } \mathbf{D} \to I, \mathrm{c}\text{: } I \vdash \mathrm{c}\text{: } \mathbf{T}}\,(\to_L)$$

$$\boxed{\mathrm{L}}\quad \dfrac{\dfrac{\dfrac{\mathrm{c} \Rightarrow \mathrm{c}\text{: } I \qquad \mathrm{c}\text{: } I, {}_{c}\mathrm{R}_{B\flat} \Rightarrow \mathrm{B}\flat : ii}{\mathrm{B}\flat\text{: } ii \vdash \mathrm{B}\flat\text{: } \mathbf{S}} \qquad \dfrac{\mathrm{A}\flat 7 \Rightarrow \mathrm{B}\flat\text{: } VII\flat}{\mathrm{B}\flat\text{:} VII\flat \vdash \mathrm{B}\flat\text{: } \mathbf{D}}}{\mathrm{B}\flat\text{: } \mathbf{S} \to VII\flat, \mathrm{B}\flat\text{: } VII\flat \vdash \mathrm{B}\flat\text{:} \mathbf{D}}\,(\to_L) \qquad \dfrac{\mathrm{B}\flat_{\mathrm{MA}}{}^{7} \Rightarrow \mathrm{B}\flat\text{: } I}{\mathrm{B}\flat\text{: } I \vdash \mathrm{B}\flat\text{: } \mathbf{T}}}{\mathrm{B}\flat\text{: } \mathbf{D} \to I, \mathrm{B}\flat\text{: } I \vdash \mathrm{B}\flat\text{: } \mathbf{T}}\,(\to_L)$$

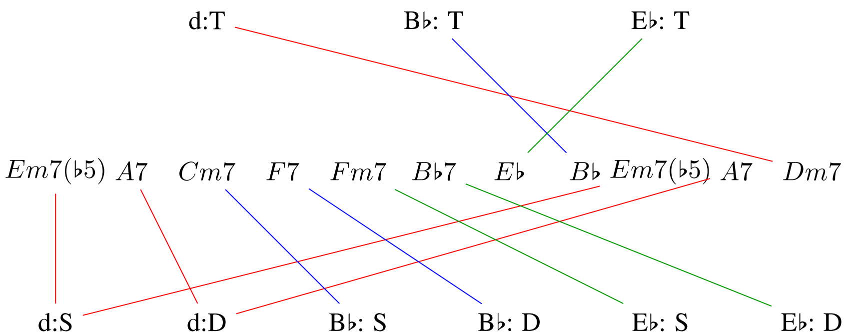
$$\boxed{\mathbf{M}}\quad \dfrac{\dfrac{\dfrac{\mathrm{Em7}^{(\flat5)}\Rightarrow \mathrm{d}^{(\flat5)}:II}{\mathrm{d}^{(\flat5)}:II\vdash \mathrm{d}^{(\flat5)}:\mathbf{S}} \qquad \dfrac{\mathrm{A7}\Rightarrow\mathrm{d}^{(\flat5)}:V}{\mathrm{d}^{(\flat5)}:V\vdash\mathrm{d}^{(\flat5)}:\mathbf{D}}}{\mathrm{d}^{(\flat5)}:\mathbf{S}\to V,\mathrm{d}^{(\flat5)}:V\vdash \mathrm{d}^{(\flat5)}:\mathbf{D}}\,(\to_L) \qquad \dfrac{\mathrm{Dm7}^{(\flat5)}\Rightarrow\mathrm{d}^{(\flat5)}:I}{\mathrm{d}^{(\flat5)}:\mathrm{I}\vdash\mathrm{d}^{(\flat5)}:\mathbf{T}}}{{}^{*(5)}\mathrm{d}^{(\flat5)}:\mathbf{D}\to I,\mathrm{d}^{(\flat5)}:I\vdash\mathrm{d}^{(\flat5)}:\mathbf{T}}\,(\to_L)$$

$$\boxed{\mathrm{N}}\quad \dfrac{\dfrac{\dfrac{{}^{*(5)}\mathrm{d}^{(\flat5)}:V\to I,\mathrm{d}^{(\flat5)}:I\vdash\mathrm{d}^{(\flat5)}:\mathbf{T} \qquad \mathrm{d}^{(\flat5)}:I,\ {}_{d^{(\flat5)}}\mathrm{R}_{c^{(\flat5)}}\Rightarrow\mathrm{c}^{(\flat5)}:II}{\mathrm{c}^{(\flat5)}:II\vdash\mathrm{c}^{(\flat5)}:\mathbf{S}} \qquad \dfrac{\mathrm{G7}\Rightarrow\mathrm{c}^{(\flat5)}:V}{\mathrm{c}^{(\flat5)}:V\vdash\mathrm{c}^{(\flat5)}:\mathbf{D}}}{\mathrm{c}^{(\flat5)}:\mathbf{S}\to V,\mathrm{c}^{(\flat5)}:V\vdash\mathrm{c}^{(\flat5)}:\mathbf{D}}\,(\to_L) \qquad \dfrac{\mathrm{Cm7}^{(\flat5)}\Rightarrow\mathrm{c}^{(\flat5)}:I}{\mathrm{c}^{(\flat5)}:\mathrm{I}\vdash\mathrm{c}^{(\flat5)}:\mathbf{T}}}{{}^{*(6)}\mathrm{c}^{(\flat5)}:\mathbf{D}\to I,\mathrm{c}^{(\flat5)}:I\vdash\mathrm{c}^{(\flat5)}:\mathbf{T}}\,(\to_L)$$

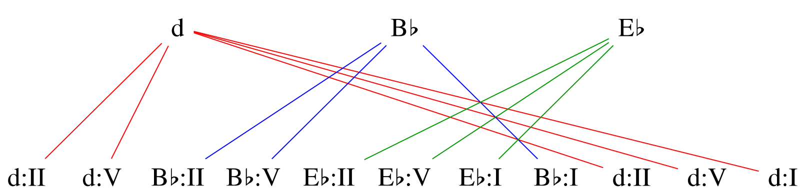
$$\boxed{\mathrm{O}}\quad \dfrac{\dfrac{\dfrac{{}^{*(6)}\mathrm{c}^{(\flat5)}:\mathbf{D}\to I,\mathrm{c}^{(\flat5)}:I\vdash\mathrm{c}^{(\flat5)}:\mathbf{T} \qquad \mathrm{c}^{(\flat5)}:I,\ {}_{c^{(\flat5)}}\mathrm{R}_{B\flat}\Rightarrow\mathrm{B}\flat\text{: }II}{\mathrm{B}\flat\text{: }II\vdash\mathrm{B}\flat\text{: }\mathbf{S}} \qquad \dfrac{F7\Rightarrow\mathrm{B}\flat\text{: }V}{\mathrm{B}\flat\text{: }V\vdash\mathrm{B}\flat\text{: }\mathbf{D}}}{\mathrm{B}\flat\text{: }\mathbf{S}\to V,\mathrm{B}\flat\text{: }V\vdash\mathrm{B}\flat\text{: }\mathbf{D}}\,(\to_L) \qquad \dfrac{\mathrm{B}\flat\Rightarrow\mathrm{G}\text{: }I}{\mathrm{B}\flat\text{: }I\vdash\mathrm{B}\flat\text{: }\mathbf{T}}}{\mathrm{B}\flat\text{: }\mathbf{D}\to I,\mathrm{B}\flat\text{: }I\vdash\mathrm{B}\flat\text{: }\mathbf{T}}\,(\to_L)$$

Tableaux for Labelled sequents

The idea of Tableux for LLS is similar to the one presented in [1], but it will be a little more intricate due to the non-linearity of harmony. For example, the first 11 chords of "Stella by Starlight" can be notated in this manner:



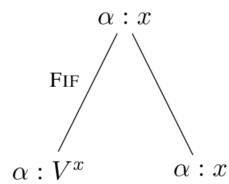
In the upper part, we have connected all the Tonic chords to their respective counterparts. In the lower part, we have established connections for the other functions, including Subdominant and Dominant chords. This comprehensive representation allows us to visualize the harmonic relationships more thoroughly and understand the interplay between different chord functions. It can alternatively be expressed in the form of chord functions:



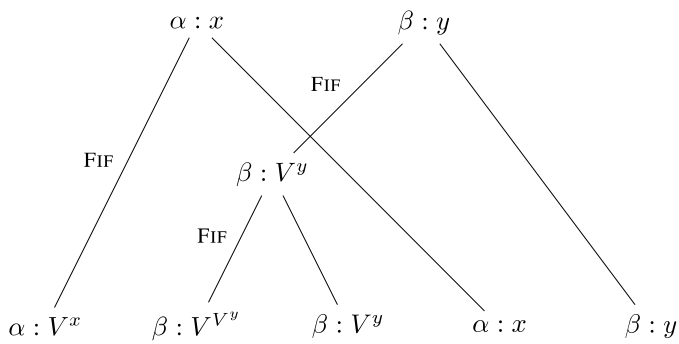
The reasons to use Tableaux are mainly twofold: one can be for a better representation of the entire structure of the harmonic sequence; it is less expressive than sequents, but it can be useful as a map. The other reason is that it can be helpful for composition; it is sufficient to create the dual of the rules introduced for LLC.

Rules

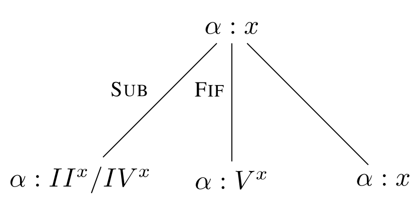
The primary challenge in reversing the rules for Tableaux lies in our intention to employ them for generating new harmonic structures. Consequently, we must adapt them to suit our specific needs. It is important to emphasize here that the rules are easily changeable, given the nature of music, which does not rigidly adhere to set rules. Let's start with a simple rule: from every chord is always possible to add a dominant, keeping in mind that we want a tonal structure. Let's call it *Fif*.



where V^x indicates the dominante of the degree x. Differently from the one presented in [1] here the dominants can be chained:

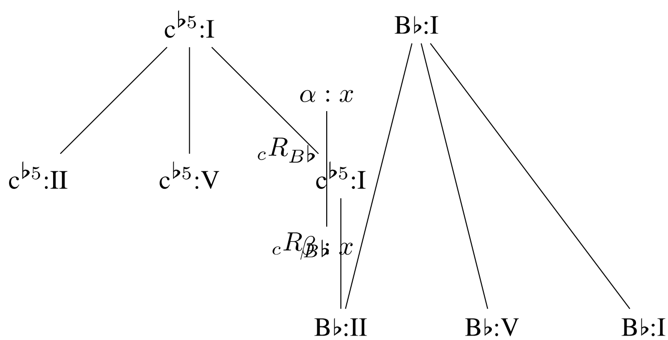


Here, we encounter three instances of the rule *Fif*, and they are intertwined. Obviously we can chain that to every α : S (α : II or α : IV), let's call it *Sub*.



Furthermore, cadences can introduce variations in the function of specific chords. For instance, a C7 chord can serve as the dominant of FMA7, yet it may also function as the first degree in a cadence that resolves to G7. An illustration of this concept can be found in the final part of Stella By Starlight, where the harmonic progression takes an unexpected turn. In general, we can write that:

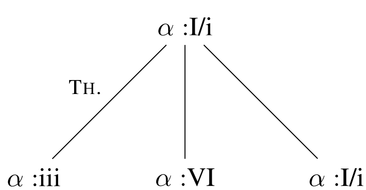
This implies that when there is an accessibility relation αRβ, we have the flexibility to change the interpretation of the chord. For instance, the final part of "Stella By Starlight," it can be seen as:



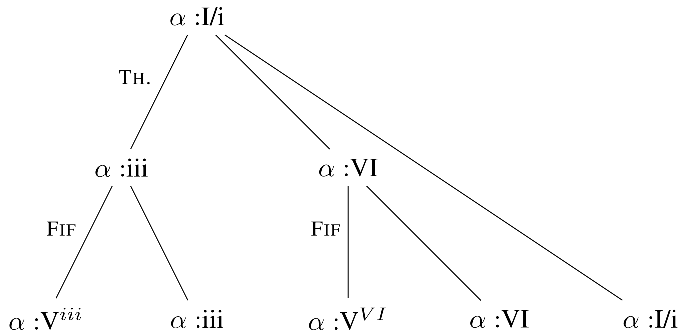
New rules

We didn't want to be complete in the list of the rules, in fact the flexibility of this system proves to be useful when incorporating new rules. For instance, if we wish to introduce a rule allowing the tonic (and only the tonic) to split into its third minor and sixth major, it can be easily formalized in the following manner, calling it *Th.*:

Obviously, these rules can be combined into new harmonic structures:



The significant aspect of this work is that such structures are inherently rational; that is, they adhere to well-established rules or rules determined by the composer. This implies that, on one hand, the structure can be very strict, while on the other hand, it can also be highly malleable.



4 Conclusions

Based upon the traditional view that the origin of language is common with music, we have regarded a sequence of chord names as a given surface sentence, and have shown a structural proof system with proposed sequent rules to decide the grammaticality. Thus far, several works have shown syntactic tree structures for the surface sequence of chords, however, in our work we have further developed the method so as to give its semantics, interpreting each chord and deriving chord functions.

Our new methodology is the labelled sequent calculus; Gentzen's sequent calculus is a strict method to show the adequacy of hierarchical composition, and thus it has been employed as a basis for Lambek calculus (LC) that is a sequent version of categorial grammar (CG). As the labelled sequent calculus has been proposed as a proof system of modal logic, we have applied the notion of label to specify the tonality or the region where each chord resides. Since a Berklee chord name can be interpreted in multiple keys, we could clarify the key with such labels, and the shift to related keys is elegantly shown by the accessibility between possible regions.

With this method, we have analyzed an actual piece of 'Stella by Starlight' and exemplified how our system works. The example is emblematic due to its unique structure; indeed, harmony is not a linear process, necessitating a non-linear analysis easily solved through labeled sequent calculus. Furthermore, we have delineated the ways to use this system in composition, leveraging the dual of sequents, i.e., Tableaux. This novel approach, initially introduced in [1], has been expanded here through various examples and thanks to labels. Labels, in fact, provided easier readability, as well as the creation of more complex harmonic structures.

However, music pieces change their styles according to era and genre, where occur many variations on the authentic theory, to tolerate dissonant chords or non-harmonic notes. Our system is malleable in this sense since we can employ different sets of sequent rules, although at the same time the adequacy of the set of rules should be verified from an objective view. Since some extra rules are rarely utilized, we should also consider the probability in the application of each rule. As for our logic, we have only applied labels for key-shift, but we should embrace the original meanings of labels, addressing the necessity ($\Box$) and the possibility ($\Diamond$), and this is our immediate future work. Another potential task is to automate this analysis system and perhaps create an automatic composer system based on the Tableaux.

Acknowledgment

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